

Calculus 1 — Practice Set & Answers

Whole-unit revision for the CAT | Limits · Differentiation · Applications · Integration · Analytical Geometry.
37 questions, every one worked.

How to use this. Do it in two passes. Pass one: the questions only, notes shut, no peeking. Pass two: mark against the answers and write down which step broke, not just which number was wrong. The broken step is what the CAT will find. If one section keeps breaking, redline it and bring me the exact line.

Section A — Limits and Continuity

1. Evaluate $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2)$.
2. Evaluate $\lim_{x \rightarrow 3} (x^2 - 9)/(x - 3)$.
3. Evaluate $\lim_{x \rightarrow \infty} (4x^3 + x)/(2x^3 + 7)$.
4. Evaluate $\lim_{x \rightarrow 0} (\sqrt{x+9} - 3)/x$.
5. Evaluate $\lim_{x \rightarrow 0} \sin(3x)/x$.
6. Let $f(x) = (x^2 - 1)/(x - 1)$ for $x \neq 1$, and $f(1) = k$. Find k so that f is continuous at $x = 1$.

ANSWERS A

1. 0/0 on substitution, so factor: $(x-2)(x+2)/(x-2) = x+2 \rightarrow 4$. **Answer 4.**
2. $(x-3)(x+3)/(x-3) = x+3 \rightarrow 6$. **Answer 6.**
3. Divide by x^3 : $(4 + 1/x^2)/(2 + 7/x^3) \rightarrow 4/2$. **Answer 2.**
4. Multiply top and bottom by the conjugate $\sqrt{x+9}+3$: top becomes $(x+9) - 9 = x$, so the fraction is $x/[x(\sqrt{x+9}+3)] = 1/(\sqrt{x+9}+3) \rightarrow 1/6$. **Answer 1/6.**
5. Write it as $3 \cdot \sin(3x)/(3x)$. Since $\lim (\sin u)/u = 1$ as $u \rightarrow 0$, the limit is $3 \cdot 1$. **Answer 3.**
6. Continuity needs the limit at 1 to equal $f(1)$. Factor: $(x-1)(x+1)/(x-1) = x+1 \rightarrow 2$. So $k = 2$. **Answer k = 2.**

Section B — Differentiation

7. Differentiate from first principles: $f(x) = 3x + 1$.
8. Differentiate from first principles: $f(x) = 1/x$.
9. Differentiate $y = 5x^4 - 3x^2 + 7x - 2$.
10. Differentiate $y = (x^2 + 1)(x^3 - 2)$.
11. Differentiate $y = (2x + 1)/(x - 3)$.
12. Differentiate $y = (3x^2 + 1)^5$.
13. Find dy/dx if $x^2 + y^2 = 25$.
14. Find the second derivative of $y = x^5 - 2x^3$.

ANSWERS B

7. $[f(x+h) - f(x)]/h = [3(x+h)+1 - (3x+1)]/h = 3h/h = 3$, constant as $h \rightarrow 0$. **$f'(x) = 3$.**

8. $[1/(x+h) - 1/x]/h = [(x - (x+h))/(x(x+h))]/h = -h/[h \cdot x(x+h)] = -1/[x(x+h)] \rightarrow -1/x^2$. **$f'(x) = -1/x^2$.**

9. Power rule term by term. **$y' = 20x^3 - 6x + 7$.**

10. Product rule: $y' = (2x)(x^3-2) + (x^2+1)(3x^2) = 2x^4 - 4x + 3x^4 + 3x^2$. **$y' = 5x^4 + 3x^2 - 4x$.**

11. Quotient rule: $y' = [2(x-3) - (2x+1)(1)]/(x-3)^2 = (2x-6-2x-1)/(x-3)^2$. **$y' = -7/(x-3)^2$.**

12. Chain rule: $5(3x^2+1)^4 \cdot (6x)$. **$y' = 30x(3x^2 + 1)^4$.**

13. Differentiate both sides with respect to x : $2x + 2y(dy/dx) = 0$, so $dy/dx = -x/y$. **$dy/dx = -x/y$.**

14. $y' = 5x^4 - 6x^2$, then $y'' = 20x^3 - 12x$. **$y'' = 20x^3 - 12x$.**

Section C — Tangents, Normals and Rates

15. Find the equation of the tangent to $y = x^3$ at $x = 2$.

16. Find the equation of the normal to $y = x^2$ at $x = 1$.

17. A particle moves so that $s(t) = t^3 - 6t^2 + 9t$ metres in t seconds. Find its velocity at $t = 2$ s.

18. Find the equation of the tangent to $y = 1/x$ at $x = 2$.

ANSWERS C

15. $y' = 3x^2$, slope = 12 at $x = 2$; point (2, 8). $y - 8 = 12(x - 2)$. **$y = 12x - 16$.**

16. $y' = 2x$, tangent slope 2, so normal slope = $-1/2$; point (1, 1). **$y - 1 = -1/2(x - 1)$** , i.e. $y = -x/2 + 3/2$.

17. $v = s'(t) = 3t^2 - 12t + 9$. At $t = 2$: $12 - 24 + 9$. **$v = -3$ m/s** (moving backwards).

18. $y' = -1/x^2$, slope at $x = 2$ is $-1/4$; point (2, $1/2$). **$y - 1/2 = -1/4(x - 2)$. $y = -x/4 + 1$.**

Section D — Maxima, Minima and Sketching

19. For $f(x) = x^3 - 3x^2 + 4$, find the stationary points and classify each with the second derivative test.

20. For $f(x) = x^3 - 3x$ on the interval $[-2, 3]$, find the absolute maximum and minimum.

21. Find the minimum value of $f(x) = x^2 - 4x + 3$.

22. Find the minimum value of $f(x) = x + 4/x$ for $x > 0$.

ANSWERS D

19. $f' = 3x^2 - 6x = 3x(x - 2)$, so $x = 0, 2$. $f'' = 6x - 6$: at $x = 0$, $f'' = -6 < 0 \rightarrow$ local max, $f(0) = 4$; at $x = 2$, $f'' = 6 > 0 \rightarrow$ local min, $f(2) = 0$. **max (0, 4), min (2, 0).**

20. $f' = 3x^2 - 3 = 3(x-1)(x+1)$, critical $x = \pm 1$. Values: $f(-2) = -2$, $f(-1) = 2$, $f(1) = -2$, $f(3) = 18$. **Absolute max 18 at $x = 3$; absolute min -2 at $x = -2$ and $x = 1$.**

21. $f' = 2x - 4 = 0 \rightarrow x = 2$; $f'' = 2 > 0$ so it is a min. **$f(2) = -1$.**

22. $f' = 1 - 4/x^2 = 0 \rightarrow x^2 = 4 \rightarrow x = 2$ (positive root). $f'' = 8/x^3 > 0$, so min. **$f(2) = 2 + 2 = 4$.**

Section E — Integration

23. $\int (6x^2 - 4x + 3) dx$
24. $\int (x^3 + 1/x) dx$
25. $\int \sin(2x) dx$
26. $\int_0^2 (3x^2 + 2x) dx$
27. $\int_1^4 (1/\sqrt{x}) dx$
28. $\int_0^\pi \sin x dx$
29. Find the area under $y = x^2$ between $x = 0$ and $x = 3$.

ANSWERS E

23. Reverse the power rule term by term. $2x^3 - 2x^2 + 3x + C$.
24. $x^4/4 + \ln|x| + C$.
25. $-1/2 \cos(2x) + C$ (check: derivative gives $\sin(2x)$).
26. $[x^3 + x^2]_0^2 = (8 + 4) - 0$. **12.**
27. $\int x^{-1/2} dx = 2\sqrt{x}$. $[2\sqrt{x}]_1^4 = 4 - 2$. **2.**
28. $[-\cos x]_0^\pi = (-\cos \pi) - (-\cos 0) = 1 + 1$. **2.**
29. $\int_0^3 x^2 dx = [x^3/3]_0^3 = 27/3$. **Area = 9 square units.**

Section F — Analytical Geometry

30. Find the distance between $(1, 2)$ and $(4, 6)$.
31. Find the equation of the line through $(2, 3)$ with slope -2 .
32. Find the centre and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.
33. Find the points where the line $y = x + 1$ meets the circle $x^2 + y^2 = 25$.
34. Find the locus of a point equidistant from $(0, 0)$ and $(4, 0)$.
35. Find the values of k for which $y = x + k$ is a tangent to $x^2 + y^2 = 25$.

ANSWERS F

30. $d = \sqrt{[(4-1)^2 + (6-2)^2]} = \sqrt{9 + 16} = \sqrt{25}$. **5 units.**
31. $y - 3 = -2(x - 2)$. **$y = -2x + 7$.**
32. Complete the square: $(x-3)^2 + (y+2)^2 = 12 + 9 + 4 = 25$. **Centre $(3, -2)$, radius 5.**
33. Substitute: $x^2 + (x+1)^2 = 25 \rightarrow 2x^2 + 2x - 24 = 0 \rightarrow x^2 + x - 12 = 0 \rightarrow (x+4)(x-3) = 0$. **Points $(-4, -3)$ and $(3, 4)$.**
34. Set the two distances equal: $x^2 + y^2 = (x-4)^2 + y^2$, which gives $x^2 = x^2 - 8x + 16 \rightarrow x = 2$. **The locus is the vertical line $x = 2$** (the perpendicular bisector).
35. For tangency, the distance from the centre $(0,0)$ to the line $x - y + k = 0$ equals the radius: $|k|/\sqrt{2} = 5 \rightarrow |k| = 5\sqrt{2}$. **$k = \pm 5\sqrt{2}$.**

Day-by-Day Revision Plan

Six days, one hour a day. Anchored to the CAT being *next week*; if the exact day is set, shift the last block forward and re-tag it as the final review.

Day	Focus	Do this	Done when
1	Limits & Continuity	Section A questions 1-6, notes shut	No 0/0 left unsolved; continuity test written out
2	Differentiation rules	Section B 7-14; redo 10, 11, 12 from blank paper	Product, quotient, chain each done without looking
3	First principles + tangents	Section B 7-8 and Section C 15-18	Can derive $f(x)=1/x$ from the definition
4	Maxima, minima, sketching	Section D 19-22 + sketch one cubic	Stationary points classified and sketch has intercepts, extrema, inflection
5	Integration	Section E 23-29	Indefinite and definite both clean; +C never forgotten
6	Analytical Geometry + mixed	Section F 30-35, then redo five you got wrong this week	Every worked answer in this set matches yours

The day before. No new problems. Redo only the ones you got wrong, then stop. Walking in warm beats cramming cold.

Built for Eshara — Calculus One CAT revision. Send back any question where the step doesn't follow and I'll take it apart with you. Same lane all semester.